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STEREO MATCHING BY HIERARCHICAL, MICROCANONICAL ANNEALING

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STEREO MATCHING BY HIERARCHICAL, MICROCANONICAL ANNEALING

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Abstract

An improved stochastic stereo-matching algorithm is presented. It incorporates two substantial modifications to an earlier version: a new variation of simulated annealing that is faster, simpler, and more controllable than the conventional "heat-bath" version, and a hierarchical, coarse-to-fine-resolution control structure. The Hamiltonian used in the original model is minimized, but far more efficiently. The basis of microcanonical annealing is the Creutz algorithm . Unlike its counterpart, the familiar Metropolis algorithm, the Creutz algorithm simulates a thermally isolated system at equilibrium. The hierarchical control structure, together with a Brownian state-transition function, tracks ground states across scale, beginning with small, coarsely coded levels. Results are shown for a 512 x 512 pair with 50 pixels of disparity.

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1 Introduction

Computational theories of vision often involve optimization, usually in an enormous state space and over a nonconvex objective function. Recently, a subtle technique from statistical physics has been used for such computations [1,2,3,4]. It is based on the physical analogy of annealing a system of molecules to its ground state, and hence is called *simulated annealing* [5,6]. To grow a perfect crystal, one starts at a high temperature and then gradually cools the substance, staying as close to equilibrium as is practical. This is directly analogous to what happens in simulated annealing: the Metropolis algorithm [7] is used to bring a synthetic system to equilibrium, and the macroscopic parameter "temperature" is used to control the rate of cooling.

The general theme is as follows:

- A particular (usually low-level) vision problem is considered.
- A representation is chosen in which the problem is modeled as an analog to a physical system of discrete "molecules." Each molecule typically corresponds to a location on a pixel lattice. The system has many degrees of freedom.
- An energy function (a Hamiltonian) is chosen that expresses constraints inherent in the problem. The solution is characterized by the *ground states* of the system; that is, those states that have the lowest energy. The ground states are difficult to specify because the state space of the system is huge — exponential in the number of molecules.
- The dynamics are simulated in a way that brings the system to the desired ground state, or at least to a very-low-energy state that approximates a ground state.

This paper presents such a model for matching stereo images. An early version is described in [3]. Two major extensions have been made that permit substantially improved performance: First, a new variety of simulated annealing is employed. It uses an simpler alternative to the standard Metropolis algorithm that is more efficient, more easily implemented, and offers more control over the annealing process. Secondly, by representing the stereo pair as a Laplacian pyramid, the system is extended to operate over several levels of resolution. It exploits relatively quickly computed minima at lower levels of resolution to initialize its state at higher levels. This method leads both to more efficiency and to the ability to deal with much larger ranges of disparities.

The basic representation remains unchanged. The state of the system encodes a dense map of discrete horizontal disparities, defined over the left image, which specify corresponding points in the right image. In the improved design, this state is relative to a particular level of resolution.

The energy of a lattice site is composed of two terms, each of which expresses a constraint important in stereo matching:

$$E_{ij} = |I_L(i,j) - I_R(i,j + D(i,j))| + \lambda |\nabla D(i,j)| \ .$$

I_L and I_R are the left and right image intensities, or some simple function of them (such as a Laplacian). Subscripts i and j range over all sites in the left image lattice. D is the disparity map. The first term is the absolute difference in intensity between corresponding points (where the correspondences are defined by the current state of the system), and the second is proportional to the local spatial variation in the current state (the magnitude of the gradient of the disparity map). The constant λ is used to balance the terms.[1] This equation expresses two competing constraints: first, the image intensities of corresponding points should be more-or-less equal, and second, the disparity map should be more-or-less continuous.

The Hamiltonian,

$$E = \sum E_{ij} \ ,$$

is unchanged; therefore, the ground states that we seek to determine or to approximate are the same. The new design is strictly concerned with improved performance. The techniques used to obtain it are not restricted to stereo matching, and should be considered for any simulated-annealing approach to low-level vision problems.

2 Microcanonical Annealing

The conventional simulated annealing algorithm uses an adaptation of the Metropolis algorithm to bring a system to equilibrium at decreasing temperatures. The Metropolis algorithm defines a Markov process that generates a sequence of states, such that the probability of occurrence of any particular state is proportional to its Boltzman weight,

$$P(S) \propto \exp(-\beta E(S)) \ ,$$

where $E(S)$ is the energy of state S and β is the inverse temperature of the system. This process generates samples from the *canonical ensemble*; that is, the system is considered to be immersed in a heat bath with a controllable temperature. Annealing is accomplished by imposing a *schedule* for reducing the temperature, $1/\beta$, so as to keep the system close to equilibrium.

Creutz has described an alternative technique that simulates the *microcanonical ensemble* [8]. In this method, the total energy remains constant (for some fixed point in the schedule). Instead of simulating a system immersed in a heat

[1] The performance of the system is not very sensitive to the value chosen for λ. In the example of Section 4, as well as all three examples in [3], $\lambda = 5$ was used.

bath, the Creutz algorithm simulates a thermally isolated system in which energy is conserved. It performs a random walk through state space, constraining states to a surface of constant energy. The simplest way to accomplish this is to augment the representation with an additional degree of freedom, called a demon, that carries a variable amount of energy, E_D. The total energy of the system is now:

$$E = E(S) + E_D \ .$$

Normally, the demon is constrained to have nonnegative energy, although the possibility of giving it negative energy is useful in annealing, as will be discussed below.

In the Metropolis algorithm a potential new state S' is chosen randomly, and is accepted or rejected based on the change in energy:

$$\Delta E = E(S') - E(S) \ .$$

If ΔE is negative, the new state is accepted; otherwise, it is accepted with probability $\exp(-\beta \Delta E)$.

The Creutz algorithm is quite similar. If ΔE is negative, the new state is accepted, and the demon energy is increased ($E_D \leftarrow E_D - \Delta E$). If ΔE is nonnegative, however, acceptance of the new state is contingent upon E_D: if $\Delta E < E_D$ the change is accepted, and the demon energy is decreased ($E_D \leftarrow E_D - \Delta E$); otherwise, the new state is rejected. Clearly, the total energy of the system remains constant.

The Creutz algorithm has several advantages. Unlike the Metropolis algorithm, it does not require the evaluation of transcendental functions. Of course, in practice these functions can be stored as tables, but we would like our algorithm to be adaptable to fine-grained parallel processors such as the Connection Machine. The small amount of local memory in such machines makes lookup tables unattractive. The Creutz algorithm can easily be implemented with only integer arithmetic — again, a significant advantage for fine-grained parallel processors and for VLSI implementation. Experiments indicate that the Creutz method can be programmed to run an order of magnitude faster than the conventional Metropolis method for discrete systems [9]. A further important advantage is that microcanonical simulation does not require high-quality random numbers.

In conventional simulated annealing, we control the process by specifying the temperature. In the microcanonical version, however, temperature is not a control parameter; it is a statistical feature of the system. In fact, standard arguments can be used to show that at equilibrium the demon energies have a Boltzman distribution:

$$P(E_D) \propto \exp(-\beta E_D) \ .$$

The inverse temperature can be determined from the mean value of the demon energy:

$$\beta = \frac{\ln(1 + 4/ < E_D >)}{4} \ .$$

Control of microcanonical annealing is accomplished by periodically removing energy from the system. The method used to generate the results in Section 4 is as follows:

1. Assume that the process begins in a random state S_0 of high energy with respect to the ground state. Call this $E_0 = E(S_0)$. The initial demon energy is zero.

2. Remove a fixed proportion of this energy, δE, by reducing the demon energy ($E_D \leftarrow E_D - \delta E$). The results of Section 4 were obtained with $\delta E = E_0/300$.

3. Run the Creutz algorithm until the system reaches equilibrium. A reasonable test for equilibrium is to consider the rate of accepted moves to states of higher energy. If the system is large enough, this rate will increase steadily until it approximates the rate of moves to states of lower energy. We terminate the process for a particular energy level when the rate of accepted moves to higher energy states decreases (measured over N site visits).

4. Repeat steps (2) and (3) until no further improvement is observed.

This procedure has only one free parameter: the ratio $\delta E/E_D$. While $E_D < 0$, the Creutz method operates as a "greedy" algorithm, accepting only moves to lower energy states and increasing E_D. When E_D becomes positive (which happens quickly if δE is small), the demon begins to exchange energy between lattice positions. As the ground state is approached, E_D remains negative because it cannot absorb more energy from the lattice.

If sites are visited in a regular scan, we observe an undesirable effect: after E_D is made negative, energy is removed from only a small area of the lattice. For example, if we visit the sites along scan lines from left-to-right, bottom-to-top, the algorithm will make "greedy" moves on the lower part of the lattice, resulting in significantly lower energy density in this area compared to the rest of the lattice. Many more scans may be required for this energy gradient to diffuse thoughout the entire lattice. This problem is easily overcome by visiting sites in random order.

The algorithm described above is sequential, and therefore is not suitable for parallel processing. Each local state transition can change E_D, which in turn can affect the next transition. Fortunately, the technique can be modified to a parallel one by using a separate demon for each lattice site. (As we add demons, the technique moves toward a canonical-ensemble simulation. In fact, if the number of demons is very large compared to the number of sites, the technique specializes to the Metropolis algorithm [8].) Preliminary experiments with one demon per site indicate good performance, although the results of Section 4 were generated with the single-demon algorithm.

3 Hierarchical Annealing

In the original model [3], annealing was performed only at the level of resolution of the stereo images. In some cases, the images were first bandpass-filtered to remove low-frequency components — in effect, a simple photometric correction for inconsistent sensor gains or film development. Lower and upper bounds on disparity were specified in advance, and all state transitions were considered with equal probability. As the range of disparity became large, this scheme required much larger amounts of computation. If we have m permissible disparities and N pixels, the size of the state space is m^N. If we double the range of disparity, the size of the state space increases by a factor of 2^N. Since N is a rather large number (2^{18} in the example shown in Section 4), we see that the state space grows explosively with increasing disparity range.

A natural extension of the method is to adopt a hierarchical, coarse-to-fine control structure. At a coarse level of resolution, the number of lattice sites (i.e., the number of pixels) and the range of disparity are small; therefore the size of the state space is relatively small.[2] We should be able to compute an approximate ground state quickly, and then use it to initialize the annealing process at the next, finer level of resolution.

Coarse-to-fine techniques have been widely used for image matching. For example, Moravec [10] used a resolution hierarchy to match discrete, point-like features. The stereo model described by Marr and Poggio [11] and further developed by Grimson [12] matched zero-crossings between hierarchies of bandpassed images. More recently, Witkin *et al.* [13] have used a continuation method to minimize an energy function through scale space. In each case, the results of low-resolution matching were used to guide the system at higher resolutions.

The Laplacian pyramid, originally developed as a compact image-coding technique [14], offers an efficient representation for hierarchical annealing. In a Laplacian pyramid, an image is transformed into a sequence of bandpass-filtered copies, $I^0, I^1, I^2, ..., I^n$, each of which is smaller than its predecessor by a factor of 1/2 in linear dimension (a factor of 1/4 in area), with the center frequency of the passband reduced by one octave. This transform can be computed efficiently by recursively applying a small generating kernel to create a Gaussian (low-passed) pyramid, and then differencing successive low-passed images to construct the Laplacian pyramid.

After constructing Laplacian pyramids from the original stereo images, disparity is reduced by a factor of 1/2 in successive levels. Therefore, at some level, disparity is small everywhere. For typical stereo images, we can take this to be level $n - 4$. [For example, if the original images were a power of 2 in linear dimension, the Laplacian images at level $n - 4$ would be 16 x 16 pixels. Disparities in the range of 0 to 63 pixels in a pair of 512 x 512 images would be reduced to the range of 0 to 1 pixel, with truncation, at the $(n-4)$th level.] We

[2] Although the state space may be large in absolute terms.

shall start annealing at this level, find an approximate ground state, and then expand the solution to the next level. To make this coarse-to-fine strategy work, however, we need two further modifications. We must use a different process for generating state transitions, and we must specify how a low-resolution result is used to start the annealing process at the next-higher level.

In the original model, the probability of choosing a new disparity for consideration as a new state was uniformly distributed over the prior range of disparities:

$$P(d_j \rightarrow d_k) = \frac{1}{m} .$$

This is not compatible with our intention of guiding the process with lower-resolution results, however. Because we are now assuming the disparity of a lattice site to be close to its correct value, a more effective generating process is to restrict the disparities to increase or decrease by one pixel:

$$P(d_j \rightarrow d_k) = \begin{cases} 0.5 & \text{if } |d_j - d_k| = 1 \\ 0 & \text{otherwise} \end{cases} ,$$

with the further restriction that the disparities are not allowed to specify corresponding points outside the boundary of the right image. In this scheme, the system undergoes Brownian motion through state space. An additional feature of this state-transition function is that it is no longer necessary to specify bounds on disparity in advance.

Expanding a low-resolution result to the next level is slightly more complicated. Obviously, one should begin by simply doubling the size of the low-resolution lattice and doubling the disparity values. Having done this, however, we find that the new state has an artificially low energy because every odd disparity value is "unoccupied," and the new map is therefore more uniform than it should be. A spurious symmetry is imposed on the new state that is solely due to the quantization of the previous result, which is likely to place the system near a metastable state (a local minimum) from which it cannot recover. Fortunately, there is an easy solution to this problem: Destroy this symmetry by adding heat. One effective way is as follows:

1. Compute the energy E_0^k of the initial state at level k. (This state has been determined by doubling the result at level $k+1$.)

2. Add a fixed proportion of this energy, ΔE^k, by increasing the demon temperature ($E_D \leftarrow E_D + \Delta E^k$). The results of Section 4 were obtained with $\Delta E^k = E_0^k/10$.

3. Run the Creutz algorithm until the system reaches equilibrium.

4. Repeat steps (2) and (3) until the number of accepted higher-energy states exceeds the number of rejected higher-energy states.

4 Results

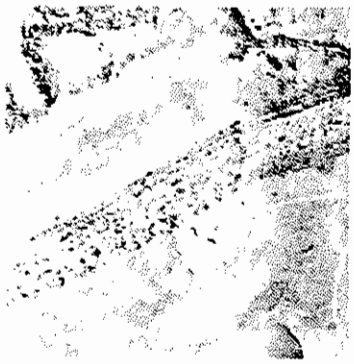
Figures 1(a) and (b) are an aerial stereo pair (512 x 512 pixels each) covering part of Martin Marietta's test site for the Autonomous Land Vehicle Project near Denver. The terrain is dominated by a long, steep "hogback" (running diagonally across the middle of the images), that separates two broad valleys. The highest terrain is in the upper left. Several roads and a few buildings may be seen. Disparity ranges from zero to 50 pixels. Figures 1(c) and (d) show the Laplacian pyramids.

The results of applying the microcanonical, hierarchical annealing algorithm to these data are illustrated in Figure 2(a), which shows the sequence of approximate ground states found at each level of the pyramid. Disparity values are displayed in 15 pseudocolors with wraparound. At the coarsest level of resolution (the tiny 16 x 16 images), the system converged to a uniform disparity map. As the system descends through the resolution hierarchy, the detail of the approximate ground states becomes increasingly precise. The disparities along the right border of the map are incorrect because the left image does not overlap the right in this region, and pixels in this region therefore have no corresponding points. Figure 2(b) is a contour map covering the upper left portion of the stereo pair, prepared by the Engineering Topographic Laboratory, and is included for comparison.

Figures 3 and 4 show some of the results in more detail. In Figure 3(a) a few points and columns have been selected from a magnified portion of the left image. The corresponding points in the right image are shown in Figure 3(b). Figure 4 is similar.

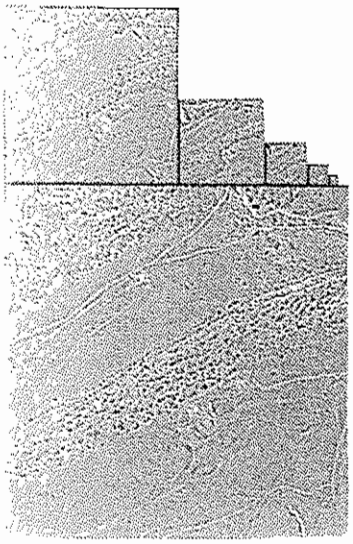
The largest problem solved by the previous version of the system was on the order of a 256 x 256 pair with 15 levels of disparity. Solving this problem required about 12 hours on a Symbolics 3600. The result shown here is a 512 x 512 pair with 50 levels of disparity, and was computed in about the same amount of time. Consider the size of the state space of the two problems. The state space of the current problem[3] exceeds the size of the old problem by a factor of more than 10^{36000}.

[3] Considering the range of disparity to be fixed at 50 pixels.

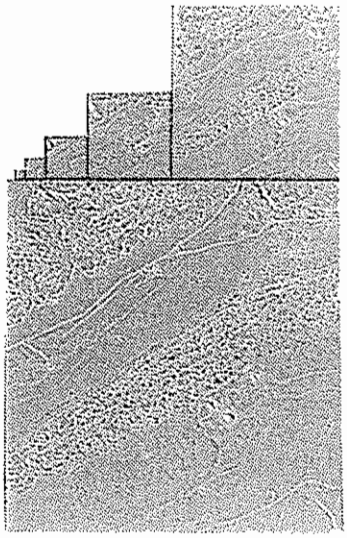


(a) Left image.

(b) Right image.

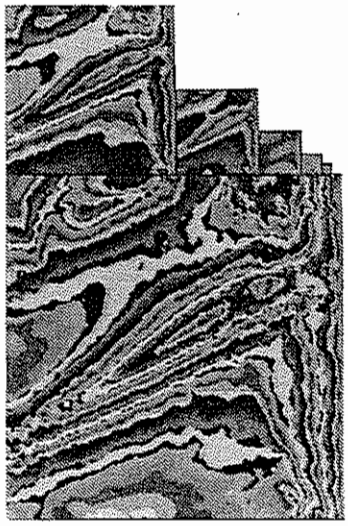


(c) Left Laplacian pyramid.

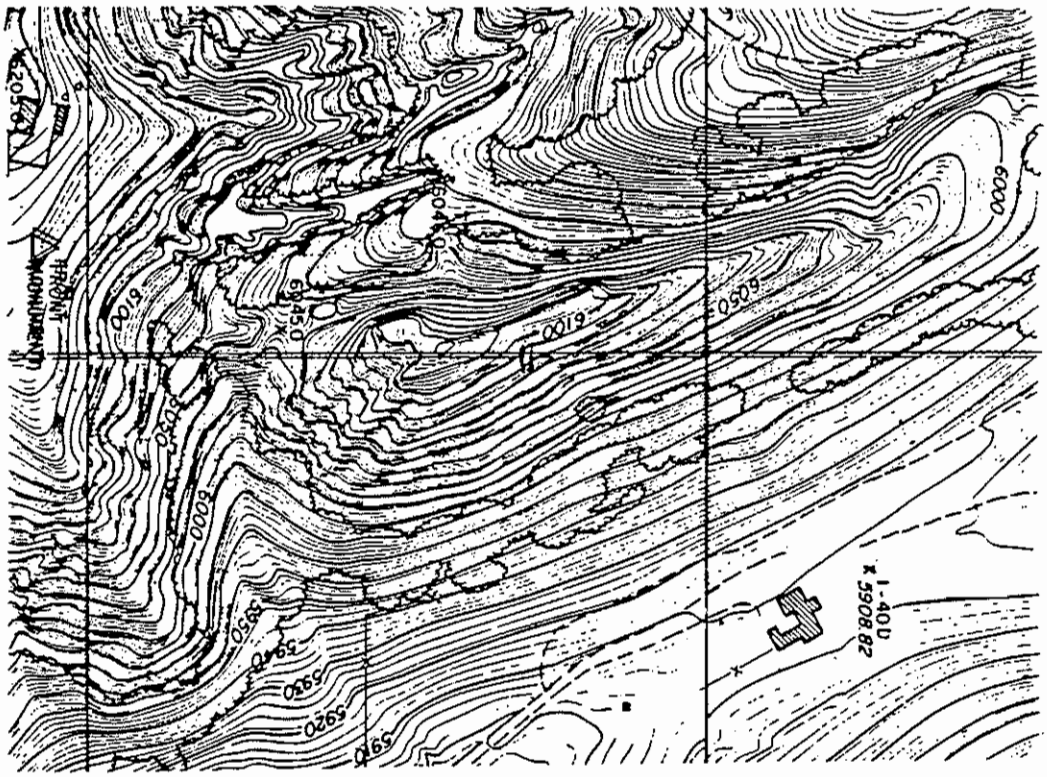


(d) Right Laplacian pyramid.

Figure 1: Aerial stereogram of the ALV test site.

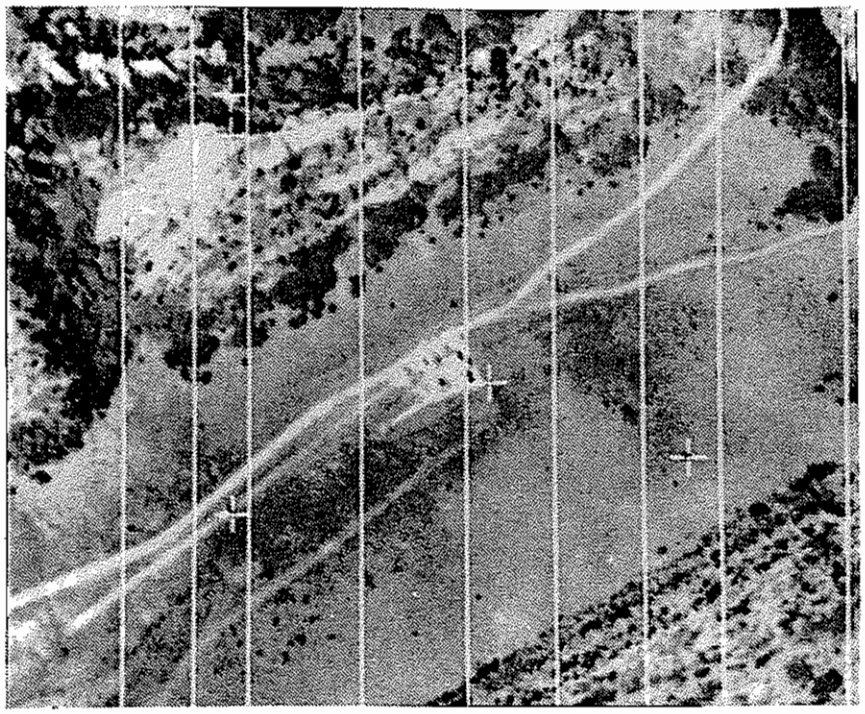


(a) Pyramid of approximate ground states.

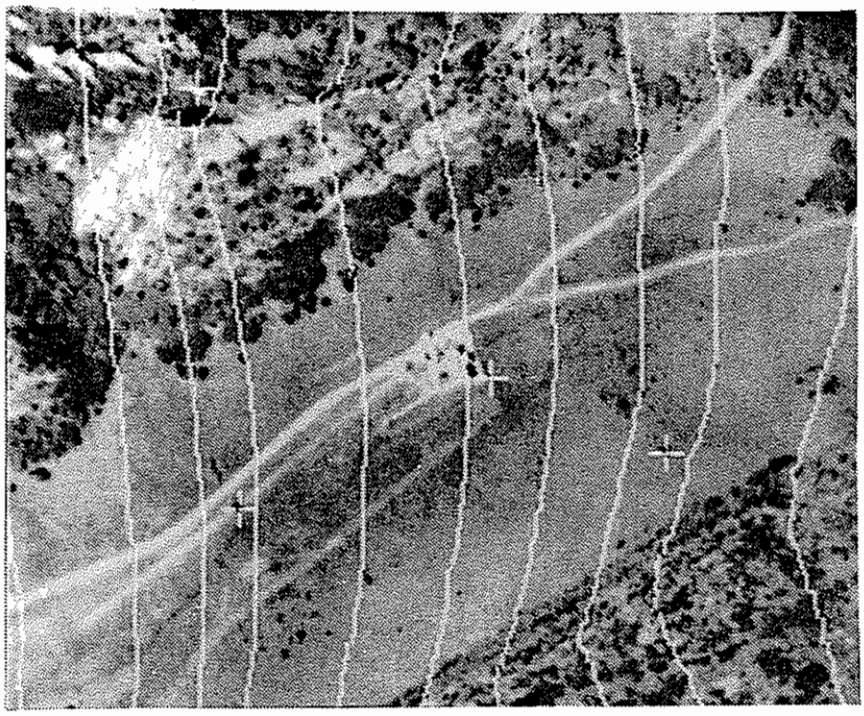


(b) Contour plot covering upper left portion.

Figure 2: Approximate ground states.

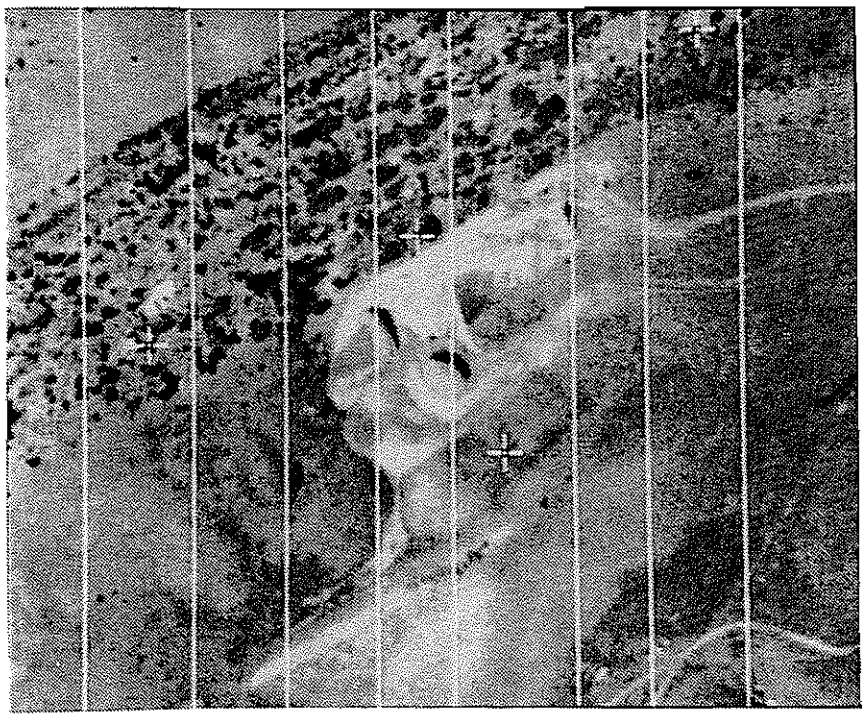


(a) Left image detail.

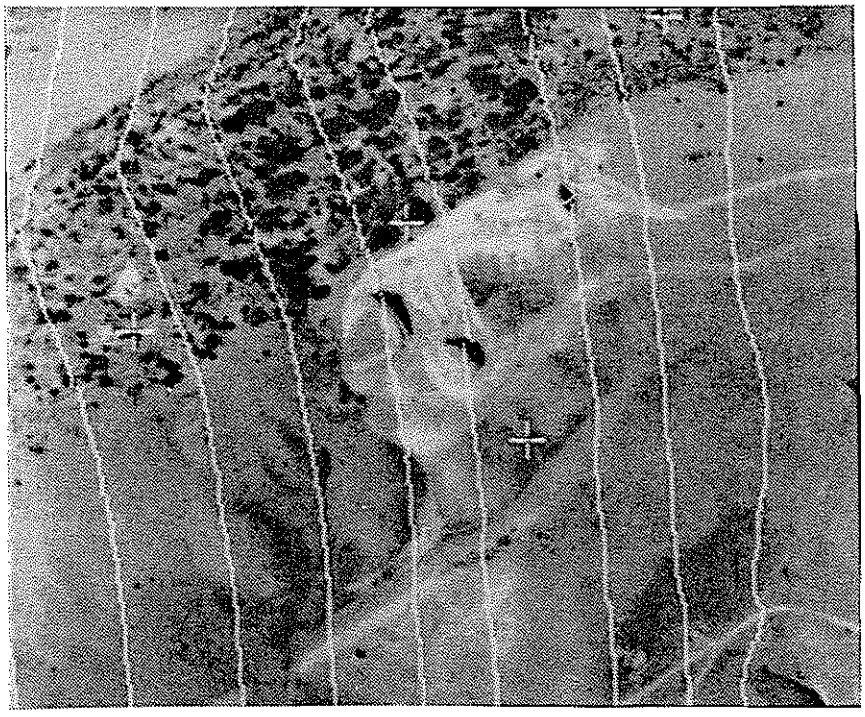


(b) Corresponding right image detail.

Figure 3: Detail A.



(a) Left image detail.



(b) Corresponding right image detail.

Figure 4: Detail B.

5 Conclusions

Two major improvements to a stochastic stereo-matching system have been described: first,a simulation of the microcanonical ensemble, as opposed to the canonical ensemble of conventional simulated annealing; and second, the extension to a hierarchical control structure based on Laplacian pyramids. Both techniques are rather general in nature and can probably be used in other simulated-annealing applications. Together, they permit solutions of stereo-matching problems that are far beyond the competence of the original system.

We are currently evaluating the quantitative increase in performance attributable to each new technique. Qualitatively, the use of microcanonical annealing yields perhaps an order of magnitude increase in efficiency. A potentially more important benefit is that its computation is simpler than the computation of standard annealing, and is therefore more readily implemented in fine-grained parallel systems. The hierarchical control structure, combined with the Brownian state-transition function, contributes most of the increased performance.

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